



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

the spectrum of Comet I., 1864*. The positions of the three bands seen by him appear to agree with those which the bright bands of this comet occupy.

This comet differs remarkably from the two small comets which I examined in the much smaller relative proportion of the light which forms a continuous spectrum. In Brorsen's comet, as it now appears, the bright middle part of the nebulosity seems to have a constitution analogous at least to that of the nucleus, and to be self-luminous; in the other comets the coma, which surrounded a distinctly marked nucleus, gave a continuous spectrum. The three comets resemble each other in the circumstance that the light of the bright central part was emitted by the cometary matter, while the surrounding nebulosity reflected solar light.

The telescopic observations of the heads of Donati's comet and of other large comets have shown that the luminous material is not at once driven off into the outer portions of the coma and the tail, but usually forms in front of the nucleus a dense luminous cloud, which for a time seems to be identical in the character of its light with that of the nucleus. It is, I believe, the outer portions only of the coma, which are frequently separated by dark spaces from the nucleus, and the tail, which the polariscope has shown to shine by reflected light.

The positions of the bands in this comet would seem to indicate a chemical constitution different from that of the nebulae, which give a spectrum of bright lines. It will be seen in the diagram that, though the brightest of the bands in the spectrum of the comet differs but little in position from the brightest line of the nebulae, the other bands are found in parts of the spectrum widely removed from those in which the other lines of the nebulae occur. The suggestion presents itself whether the broad, nebulous bands may not indicate conditions of temperature and molecular state different from those which occur in the gaseous nebulae. Plücker has shown that nitrogen and some other substances give totally different spectra, under different conditions of temperature and tension. The spectrum of this comet, however, does not resemble the other spectrum of nitrogen, which Plücker distinguishes as the spectrum of the first order†.

IX. "Mémorial on 'Undevelopable Uniquadric Homographies.'" By MARTIN GARDINER, C.E. Communicated by the Rev. R. TOWNSEND, F.R.S. Received April 13, 1868.

(Abstract.)

In this paper the author's method of investigation is purely geometrical throughout, its arrangement of details is systematic and natural, and it is divided into eight chapters, the first seven of which are preparatory to the consideration of the interesting problem discussed at length in the eighth,

* Philosophical Transactions, 1864, p. 158.

† Ibid. 1865, p. 9.

the direct, general, and complete solution of which is claimed to be given in it for the first time.

Chapter I., after some preliminary general properties, resulting immediately from the known properties of homographic systems of points on plane conics, treats more particularly of the simplest case of such systems on undevelopable quadrics, viz. of the case of systems in perspective, the several pairs of whose corresponding constituents possess manifestly the property of interchangeability; shows that systems having three double points not in the same tangent plane to the quadric on which they lie are necessarily of that class, except only when they have a fourth double point not in the plane of the other three, in which case they altogether coincide; and gives simple instances in which the problem, whose solution is the principal object of the memoir, is manifestly either "wholly or partially porismatic," as he terms it.

Chapter II. treats of systems whose several pairs of corresponding points are interchangeable but which are not in perspective; shows that their several chords of connexion intersect the same two reciprocal lines with respect to the quadric on which they lie; shows the relation between either system and the perspective of the other to any point on either of those lines, also the relation between either of two systems in perspective and the perspective of either to any point conjugate to their centre of perspective with respect to the surface; and shows how to construct the two reciprocal lines from two pairs of corresponding points of the systems.

Chapter III. treats of systems whose several pairs of corresponding points connect through a single common line, which have therefore an infinite number of double planes passing through that line; shows that their several chords of connexion, besides intersecting the line, all touch a second quadric having double contact with the original, both at its two points of intersection with the line, and also at its two points of intersection with the reciprocal line; proves a property of the cone enveloping either surface from any vertex taken arbitrarily on either line; shows the relation between either system and the perspective of the other to any point on either line; and shows how to construct the two reciprocal lines from two pairs of corresponding points of the systems.

Chapter IV. treats of systems having two of their four double planes non-tangential to the quadric on which they lie; shows that the chords connecting their several pairs of corresponding points touch two cones enveloping the quadric along two planes colinear with and harmonically conjugate to each other with respect to the two non-tangential double planes; proves that those touching along a plane section of either cone generate a skew quadric; shows that the two homographic systems determined by the two correspondents in the two systems of a variable point on the quadric are of the class considered in the preceding chapter, having an infinite number of double planes passing through the line of intersection of the two non-tangential double planes of the original systems; and shows the

relation between either system and the perspective of the other to any point on that line of intersection.

Chapter V. treats of systems having their four double planes all tangential to the quadric on which they lie, shows that the chords connecting their several pairs of corresponding points touch two other quadrics having quadruple contact with the original at the four double points of the systems, and gives various constructions for the determination of the four double points when the law connecting the several pairs of corresponding points of the systems is given or known.

Chapter VI. gives various criteria for determining in certain cases to which of the preceding classes two homographic systems belong, where, as in the problem whose solution forms the principal object of the memoir, the law connecting the same pairs of corresponding points of the systems is given or known.

Chapter VII. contains numerous theorems, several of much interest and originality, respecting open and closed polygons inscribed in undevelopable quadrics, whose sides pass in the same order of sequence through a common system of points in space, all deduced from the principles established in the preceding chapters, and several having direct reference to the interesting problem to be considered in the next and closing chapter.

Chapter VIII.—Given an undevelopable quadric and n fixed points in to find the space; first extremities of inscribable closed n 'gons, or the locus of the first extremities when the inscription of the closed n 'gons is porismatic.

When the number n of given points is odd.

Assume any three points a, b, c in the surface, no two of which are on one generator, as first extremities, and proceed to inscribe $2n$ 'gons.

(1) If the three points be found to be first extremities of closed n 'gons, then will the trace of their plane be the locus of first extremities of closed n 'gons, the problem in such case being partially porismatic.

(2) If the points are first extremities of closed $2n$ 'gons, or if two of them be first extremities of closed $2n$ 'gons and the third one a first extremity of a closed n 'gon, or if one of the points be the first extremity of a closed $2n$ 'gon, and the other two points first extremities of closed n 'gons, then the line or lines forming the closing chords of the open n 'gons composing the $2n$ 'gon or $2n$ 'gons (as may be) and the tangent plane or planes at the first extremity or extremities of the closed n 'gon or n 'gons (as may be) meet in one point ρ , the trace of whose polar plane R is the locus of first extremities of inscribable closed n 'gons, the problem in such case being partially porismatic.

(3) If two of the points be first extremities of closed n 'gons and the third point the first extremity of an open $2n$ 'gon, then the problem is non-porismatic, and the two closed n 'gons are the only inscribable closed n 'gons. Moreover the reciprocal of the line joining the first extremities of the two closed n 'gons will pierce the quadric in points which are the first extremi-

ties of inscribable closed $2n'$ gons (real or imaginary according as the quadric S is ruled or convex).

(4) If one of the three chosen points be the first extremity of an open $2n'$ gon (no matter as to the other two points) the problem is non-porismatic, and we can find the first extremities of the closed n' gons by either of the four following methods:—

First method.—Continue the $2n'$ gon until a $4n'$ gon be formed, and draw the plane P which contains the extremities of this $4n'$ gon and the point of junction of the two open $2n'$ gons composing it. Assume another point in the surface, not in the trace of the plane P , and, making it a first extremity, inscribe another $4n'$ gon; and through the extremities of this $4n'$ gon and the point of junction of the two open $2n'$ gons composing it draw a plane Q . Then with the line xx of intersection of the planes P and Q pierce the quadric in the only points (real or imaginary as may be) which are first extremities of closed n' gons; and the line ii reciprocal to xx will pierce the quadric in points (real or imaginary as may be) which are first extremities of closed $2n'$ gons.

Second method.—By the additional inscription of another open n' gon convert the open $2n'$ gon into an open $3n'$ gon, and put A, B, C to represent the three open n' gons composing the open $3n'$ gon. Find the point of puncture of the line through the first extremity of A and the final extremity of B with the tangent-plane at the junction of A and B ; find the point of puncture of the line through the first extremity of B and the final extremity of C with the tangent-plane at the junction of B and C . Then will the line xx through the two points of puncture pierce the quadric in two points (real or imaginary as may be) which are the first extremities of the only inscribable closed n' gons; and the line ii , which is reciprocal to xx , will pierce in first extremities of closed $2n'$ gons.

N.B. When S is a hyperboloid of one sheet, and that the first extremities of the closed n' gons are *real*, then the first extremities of the closed $2n'$ gons are also *real*; and it is evident there are two pairs of generators which are corresponding interchangeable lines in the homographic figures in which the extremities of the inscribable n' gons are pairs of corresponding points. It is moreover evident that when all the corresponding points of such figures are not interchangeable, these are the only pairs of interchangeable generators; and we must not assume the first extremities of the $4n'$ gons or $3n'$ gons in these lines.

Third method.—Let $o_1, o_2, o_3, \dots o_n$ be the n fixed points through which the sides must pass in order.

Assume the constant homological ratio -1 for homological systems, and making o_1 vertex and its polar plane axis, find the point α_1 homological to the centre α_0 of the quadric; assume o_2 as vertex and its polar plane as axis, and find the point α_2 homological to α_1 ; and proceed thus directly in order through the n points until arrived at the point α_n . Assume o_n as vertex and its polar plane as axis, and find the point α_{n-1} homological to the

centre α_0 of the quadric; assume α_{n-1} as vertex and its polar plane as axis, and find the point α_{-2} homologous to α_{-1} ; and proceed thus in reverse order through the rest of the n given points until arrived at the point α_{-n} (the points α_n, α_{-n} will be distinct). Draw the diametral plane A_0 which bisects the chords parallel to the line $\alpha_n \alpha_{-n}$ (it will also bisect $\alpha_n \alpha_{-n}$); assume any two points in the trace of this plane as first extremities, and inscribe two n' gons in the quadric; through the point α_n and the final extremities of these n' gons draw the plane A_n ; find the line of intersection ii of the planes A_0, A_n , and its reciprocal xx . Then will xx always pierce in the two points (real or imaginary as may be) which are the first extremities of the only inscribable closed n' gons; and the line ii will pierce in first extremities of closed $2n'$ gons.

Fourth method.—Find the points α_{-n} and α_n as in last method; assume any point α_0 in the surface as first extremity, and inscribe a n' gon whose last extremity we may represent by α_n ; draw the plane D_1 which contains the line $\alpha_{-n} \alpha_0$ and the point α_0 ; draw the plane D_2 which contains the $\alpha_n \alpha_0$ and the point α_n ; in the lines $\alpha_0 \alpha_{-n}, \alpha_0 \alpha_n$ find the points m_1, m_2 such that

$$\frac{\alpha_{-n} m_1}{\alpha_0 m_1} = \frac{\alpha_n m_2}{\alpha_0 m_2} = + \sqrt{\frac{\alpha_{-n} D_1 \cdot \alpha_n D_2}{\alpha_0 D_1 \cdot \alpha_0 D_2}};$$

and in the same lines find the points h_1, h_2 such that

$$\frac{\alpha_{-n} h_1}{\alpha_0 h_1} = \frac{\alpha_n h_2}{\alpha_0 h_2} = - \sqrt{\frac{\alpha_{-n} D_1 \cdot \alpha_n D_2}{\alpha_0 D_1 \cdot \alpha_0 D_2}}.$$

Put Σ_p, Σ_{ii} to represent the homographic figures in which the first and final extremities of inscribable n' gons are corresponding points. Regard m_2 and h_2 as points in Σ_p , and find their correspondents m_3, h_3 in Σ_{ii} ; draw the planes $m_1 m_3, h_1 h_3$, and find their line of intersection xx and its reciprocal ii . Then will the line xx pierce the quadric in the only points (real or imaginary as may be) which are first extremities of inscribable closed n' gons, and the line ii will pierce in first extremities of closed $2n'$ gons.

Moreover the planes $m_1 m_3, h_1 h_3$ are the only two double planes of the figures Σ_p, Σ_{ii} which are non-tangential to the quadric.

N.B. When the points α_{-n}, α_n are coincident, the inscription of the closed n' gons is partially porismatic, and one of the two points which divide $\alpha_{-n} \alpha_n$ and the diameter coincident with it harmonically is the point of concurrence of the closing chords of all inscribable open n' gons, and the polar plane of which passes through the other, &c.

When the centre of the quadric is a double point, then according as all the closing chords are parallels or pass through the centre, so will the locus of the first extremities of the closed n' gons be the trace of a diametral plane or of a plane at infinity.

When the number n of given points is even.

Assume three points a_1, b_1, c_1 on the surface, no two of which are on

the same generator ; and, making them first extremities, proceed to inscribe $2n'$ gons.

(1) If these assumed points be found to be first extremities of closed n' gons, the problem is fully porismatic, and every point in the surface will be the first extremity of an inscribable closed n' gon.

(2) If two of the points be found to be first extremities of closed n' gons and the other not, then the line xx through these two points, and the line ii reciprocal to xx , pierce the quadric in four points (the punctures made by ii being real or imaginary as may be) which constitute the first extremities of all the inscribable closed n' gons.

(3) If one of the points be the first extremity of a closed n' gon, and another of them be first extremity of a closed $2n'$ gon. Draw the closing chord of the two open n' gons composing the closed $2n'$ gon to pierce the tangent-plane at the first extremity of the closed n' gon in the point ρ ; in the closing chord find the point μ which is conjugate to the point of puncture ρ ; through μ and the first extremity of the closed n' gon draw the line xx , and find the line ii reciprocal to xx . Then will the lines xx and ii pierce the quadric in the four points which constitute the first extremities of all the inscribable closed n' gons.

(4) If two of the assumed points be found to be first extremities of closed $2n'$ gons, we may find the first extremities of the inscribable closed n' gons by either of the four following methods :—

First method.—Draw the two closing chords of the open n' gons composing the $2n'$ gons ; and, if these chords intersect in a point ρ , draw the line xx which is polar of ρ in respect to the trace of the plane of the two chords ; find the line ii reciprocal to xx . Then will xx and ii pierce the quadric in the first extremities of the closed n' gons. But if the chords do not intersect, draw tangent-planes at their extremities ; find the two pairs of points (one pair in each chord) which divide these closing chords and the segments intercepted by the tangent-planes harmonically ; draw the line xx through the two of these points which divide the chords internally ; draw the line ii through the two points which divide the chords externally. Then will xx and ii be reciprocal lines piercing the quadric in four points which are first extremities of the inscribable closed n' gons.

Second method.—Find a_1a_2 , b_1b_2 , c_1c_2 the closing chords of three open n' gons. If any two of these intersect, proceed as in the last method ; but if not, proceed as follows :—In the chord a_1a_2 find the point m which corresponds to infinity (on the same line) in one of the homographic figures in which the extremities of all inscribable n' gons are corresponding interchangeable points ; in the same chord a_1a_2 find the two points x and i such that $mx=mi=\sqrt{ma_1 \cdot ma_2}$; through x draw the line xx which cuts the two non-planar chords b_1b_2 , c_1c_2 ; through i draw the line ii which cuts the same two non-planar chords b_1b_2 , c_1c_2 . Then will xx and ii be reciprocal lines piercing the quadric in the four points which are the first extremities of closed n' gons.

Third method.—Draw a_1a_2 , b_1b_2 the closing chords of the open n 'gons composing the closed $2n$ 'gons. If these two lines intersect in a point ρ , draw the line xx through the points in a_1a_2 , b_1b_2 which are harmonic conjugates to ρ in respect to the segments a_1a_2 , b_1b_2 ; find the line ii reciprocal to xx . Then will xx and ii pierce the quadric in the four points which are first extremities of closed n 'gons. But if a_1a_2 , b_1b_2 do not intersect, then find the final extremity c_2 of the n 'gon having the point c_1 as first extremity; and if the line c_1c_2 cuts either a_1a_2 or b_1b_2 , the lines xx and ii can be found in the manner just indicated. If c_1c_2 do not cut either of the lines a_1a_2 , b_1b_2 , then find the points α_1 , α_2 in which a_1a_2 pierces the planes $b_1b_2c_1$, $b_1b_2c_2$; find the points β_1 , β_2 in which b_1b_2 pierces the planes $a_1a_2c_1$, $a_1a_2c_2$; find the points h_1 , h_2 which divide the segments a_1a_2 , $\alpha_1\alpha_2$ harmonically; find the points k_1 , k_2 which divide the segments b_1b_2 , $\beta_1\beta_2$ harmonically; through the two points h_1 , h_2 , cutting a_1a_2 , b_1b_2 internally, draw the line xx ; through the two points k_1 , k_2 , cutting a_1a_2 , b_1b_2 externally, draw the line ii . Then will xx and ii be reciprocal lines piercing the quadric in the four points which are first extremities of closed n 'gons.

Fourth method.—Put Σ_i and Σ_{ii} to represent the homographic figures in which the first and final extremities of all inscribable n 'gons are corresponding points; find the point α_1 which corresponds in either of the figures Σ_i , Σ_{ii} to the centre α_0 of the quadric regarded as belonging to the other figure; find the diameter d_1d_2 which contains the points α_0 , α_1 ; find the points p , q which divide the segments d_1d_2 and $\alpha_0\alpha_1$ harmonically; draw the plane P which is polar to the point p which lies outside the quadric; find the point a_2 which is final extremity of a n 'gon whose first extremity is in the trace of the plane P; draw xx the diameter of the trace of P which bisects a_1a_2 (the point a_2 will be in the trace of P); find the line ii reciprocal to xx . Then will xx and ii pierce in first extremities of the four inscribable closed n 'gons. But if the centre α_0 of the quadric be a double point of the figures Σ_i , Σ_{ii} , proceed as follows:—Inscribe any n 'gon in the quadric, and draw the diameter xx which bisects its closing chord. Then will the diameter xx and its reciprocal at infinity pierce the quadric in the four points which are first extremities of closed n 'gons.

(5) When we can inscribe an open $2n$ 'gon the problem is always non-porismatic, and we can find the lines xx , ii which pierce the quadric in first extremities of the four closed n 'gons by either of the four following methods:—

First method.—Put Σ_i and Σ_{ii} to represent the homographic figures in which the first and final extremities of all inscribable n 'gons are corresponding points. In the figures Σ_i and Σ_{ii} find the points o_1 and o_2 which are the correspondents of the centre o of the quadric regarded as belonging to the figures Σ_i and Σ_{ii} ; draw the diameter ror which bisects o_1o_2 ; find the line $r_1o_1r_1$ in Σ_i which corresponds to ror in Σ_{ii} ; bind the line $r'r'$ reciprocal to r_1r_1 ; through the centre o draw the diametral plane K which bisects all chords parallel to ror ; find the points ρ_1 , ρ' in which the reciprocal lines r_1r_1 ,

$r'r'$ pierce the plane K ; draw any plane P parallel to K , and through the points μ_1, μ' in which it cuts r, r_1 and $r'r'$ draw lines parallel to the diameter rr to pierce the plane K in points ϕ_1, ϕ' ; through the centre o draw (see 'Section of Ratio' of Apollonius) the two lines ox_1x', oi_1i' so that

$$\rho_1 x_1 : \rho' x' :: \rho_1 i_1 : \rho' i' :: \rho_1 \phi_1 : \rho' \phi';$$

through x_1 and x' draw lines parallel to rr to cut $r_1 r_1$ and $r'r'$ in points x and x ; through i_1 and i' draw lines parallel to rr to cut $r_1 r_1$ and $r'r'$ in i and i . Then will the lines xx and ii be reciprocals piercing the quadric in the four points which are first extremities of the inscribable closed n' 'gons.

Second method.—In the closing chord $c_1 c_2$ of any inscribed open n' 'gon assume any point p , and for the moment regard it as $n+1$ th point of a series having the n fixed points as first n points; choose four points b_1, d_1, e_1, f_1 in the surface so that no two of the five c_1, b_1, d_1, e_1, f_1 lie on one generator; find the final points b_2, d_2, e_2, f_2 of inscribed $(n+1)$ 'gons whose sides pass in order through the $n+1$ points, and which have b_1, d_1, e_1, f_1 as first extremities. Then (representing tangent-planes at points by capital letters of like names, and subscript numbers as the small ones representing the points of contact) in the chord $d_1 d_2$ find the point d_3 such that

$$\frac{d_1 d_3}{p_2 d_3} = \frac{d_1 B_1}{d_2 B_2} : \frac{c_1 B_1}{c_1 B_2};$$

in the chord $e_1 e_2$ find the point e_3 such that

$$\frac{e_1 e_3}{e_2 e_3} = \frac{e_1 B_1}{e_2 B_2} : \frac{c_1 B_1}{c_1 B_2};$$

in the chord $f_1 f_2$ find the point f_3 such that

$$\frac{f_1 f_3}{f_2 f_3} = \frac{f_1 B_1}{f_2 B_2} : \frac{c_1 B_1}{c_1 B_2}$$

(the points d_3, e_3, f_3 must be so determined as that a *real* tangent-plane can pass through either of them); draw the plane $d_3 e_3 f_3$ and it will touch the quadric in a point a_1 ; draw the line $c_1 a_1$, and find the point q in it which is conjugate to p . Now if we regard the n given fixed points and the point q as the $n+1$ points of a series, every point in the surface will be the first extremity of a closed $2(n+1)$ 'gon. Find (by last case) the two reciprocal lines yy, zz which pierce the quadric in first extremities of closed $(n+1)$ 'gons whose sides pass in order through these $n+1$ points; find the line $p'q'$ reciprocal to pq ; draw the lines ax, ii , each of which cuts the four non-planar lines $yy, zz, pq, p'q'$. Then will ax and ii be reciprocal lines piercing in four points which are the first extremities of the inscribable closed n' 'gons.

Third method.—On the closing chord $c_1 c_2$ of any inscribed open n' 'gon assume any point p , and regard it for the moment as the $n+1$ th point of a series to which the n fixed points belong. Assume three points b_1, d_1, e_1 in the surface so that no two of the points c_1, b_1, d_1, e_1 are on one generator, and find the final extremities b_2, d_2, e_2 of inscribed n' 'gons having b_1, d_1, e_1 as first extremities; draw the tangent-plane C_1 at the point c_1 ; put $D_1, D_2,$

E_1, E_2 to represent the four planes $d_1b_1c_1, d_2b_2c_1, e_1b_1c_1, e_2b_2c_1$ respectively; through the line of intersection of the planes D_1, D_2 draw the plane P the distances of any point in which from D_1 and D_2 have to each other the ratio of $\frac{b_1D_1}{b_2D_2}$ to $\frac{b_1C_1}{b_2C_1}$; through the line of intersection of the planes E_1, E_2 draw the plane Q which is such that the ratio of the distances of any point in it from E_1 and E_2 is the same as that of $\frac{b_1E_1}{b_2E_2}$ to $\frac{b_1C_1}{b_1C_1}$; find the line of intersection mm of the planes P, Q (this line mm will be a tangent to the quadric), and the point a_1 in which it touches the quadric; in the line c_1a_1 find the point q conjugate to p . Then if we regard the n given fixed points and the point q as $n+1$ points of a series, any point in the surface will be first extremity of a closed $2(n+1)$ 'gon. Find, by the preceding case, the two reciprocal lines yy, zz which pierce the quadric in first extremities of closed $(n+1)$ 'gons whose sides pass in order through these $n+1$ points; find the line $p'q'$ reciprocal to pq ; draw the lines xx and ii each of which cuts the four non-planar lines $yy, zz, pq, p'q'$. Then will xx and ii be reciprocals piercing in first extremities of the four inscribable closed n' 'gons.

Fourth method.—The following method is applicable in all cases in which n is even. Omit temporarily the n th point o_n of the given n points, and find the line U which pierces the quadric in the two first extremities of inscribable closed $(n-1)$ 'gons whose sides pass in order through the $n-1$ points; find the point q in the line U which is conjugate to the omitted n th point o_n . Then if we regard the $n-1$ given points and the point q as forming the n points of a new series, any point in the surface will be the first extremity of a closed $2n'$ 'gon. Find the two reciprocal lines yy, zz which pierce the quadric in first extremities of closed n' 'gons whose sides pass in order through the new series of n points; find the line $p'q'$ reciprocal to o_nq ; draw the lines xx, ii each of which cuts the four non-planar lines $yy, zz, o_nq, p'q'$. Then will the lines xx and ii be reciprocals piercing the quadric in first extremities of the four inscribable closed n' 'gons whose sides pass in order through the n given points. But if the inscription of the closed $(n-1)$ 'gons be partially porismatic, and ρ the point of concurrence of the closing chords of the inscribable open $(n-1)$ 'gons, then will the line xx through o_n and ρ , and the line ii reciprocal to xx pierce the quadric in the first extremities of the inscribable closed n' 'gons.

N.B. And if o_n be in such case coincident with ρ , then the problem is fully porismatic, and every point in the surface is the first extremity of a closed n' 'gon.

N.B. We may also observe that when the inscription of the closed $(n-1)$ 'gons is non-porismatic, and that the point o_n is situated in the line U , then, by conceiving q coincident with o_n the lines yy, zz will be identical with xx and ii .

I may observe that the general problem can be completely solved by

"methods of reduction," amongst which the following is perhaps the most obvious and simple :—

Let S be the quadric, and $o_1, o_2, o_3, o_4, \dots o_n$ the n given points. Put xx for the line through o_1 and o_2 . Instead of o_2 and o_1 we can substitute the point p_2 in which the line xx is cut by the plane $o_3o_4o_5$, and another point p_1 determinable in the same line xx . Then instead of the four planar points p_2, o_3, o_4, o_5 we can (see theorem 38) substitute two other points p_3, p_4 in the same plane; and therefore instead of the series of n points, we can substitute the series of $n-2$ points $p_1, p_3, p_4, o_6, o_7, \dots o_n$ and the inscribable $(n-2)$ 'gons, closed and open as may be, whose sides pass in order through these points will have extremities identical with the extremities of inscribable n 'gons. And thus step by step we can reduce the number of sides, until at length we find three points or four points, according as n is odd or even, such that the extremities of all inscribed 3'gons or 4'gons whose sides pass in order through such points are identical with extremities of inscribable n 'gons whose sides pass through the original n points; and therefore to solve the problem all we have to do is to inscribe the closed 3'gons or closed 4'gons as may be.

And in respect to this method we may observe,—

(1) If any four consecutive points of any of the series be colinear and such as to render the inscription of closed 4'gons *real*, we may omit such points altogether from the series.

(2) When n is *odd*, and that we reduce the problem to the inscription of closed 3'gons whose sides pass through three known points, then should such points be colinear or form a conjugate triad, the problem will be partially porismatic.

(3) In the case in which n is *odd*, it is easy to perceive how the problem can be reduced to the drawing of a line through a known point to cut two reciprocal lines (which *point* will be on *one* of the lines when the problem is partially porismatic). And when n is *even*, it is easy to see how the problem can be reduced to the drawing of the two lines which cut two pair of (determinable) reciprocal lines.

(4) The following method of finding the line in the plane of four points which pierces in first extremities of closed 4'gons is obvious :—Let o_1, o_2, o_3, o_4 be the four planar points.

Find p the point of intersection of the lines o_1o_3, o_2o_4 ; in the line o_1o_2 find the point m such that o_1o_2, mp , and the pair of (real or imaginary) points in which o_1o_2 pierces the quadric, will form an involution; in the line o_3o_4 find the point n such that the pairs of points o_3o_4, pn , and the points in which o_3o_4 pierces the quadric, form an involution. Then will the line mn pierce in first extremities of closed 4'gons.